CORRECTION

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Correction to the author's paper, "The Penetration of Rigid Punch into a Plastic Half-Space", (*PMM Vol.22, No.2, 1959*)

(Ispravlenie k rabote avtora, "O vdavlenii zhestkikh shtampov v plasticheskoe poluprostranstvo")

In [1] basic relations were considered in the theory of spherical state of deformation. In deriving some particular integrals the author did not take into account the facts that, contrary to the case of the theory of plane strain of an ideally plastic body, where it is possible to find finite relationships along the characteristic lines, and, consequently, to demonstrate that the characteristic lines can be used as curvilinear coordinates, in the theory of the spherical state of deformation the corresponding relationships are generally non-integrable. This circumstance led to an erroneous conclusion that the limiting pressure resulting from the penetration of a wedge-like punch into a half-space remains constant.

The results in [1] determine completely a canonical system of equations which govern the relationships along the characteristics in the plane of two variables. Thus, the problems of spherical state of deformation can be solved by the finite differences method. Below a numerical solution is given of a problem of the penetration of a wedge-like punch into a half-space.

The equations of the theory of a spherical state of deformation are, [1]:

$$\frac{\partial\sigma_\theta}{\partial\theta}+\frac{1}{\sin\theta}\frac{\partial\tau_{\theta\varphi}}{\partial\varphi}+(\sigma_\theta-\sigma_\varphi)\cot\theta=0 \qquad (1)$$

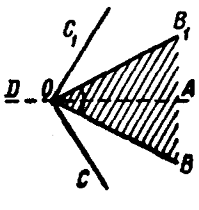

Fig. 1.

$$\frac{\partial \tau_{\theta\varphi}}{\partial \theta} + \frac{1}{\sin\theta}\frac{\partial \sigma_{\varphi}}{\partial \varphi} + 2\tau_{\theta\varphi}\cot\theta = 0$$
$$(\sigma_\theta - \sigma_\varphi)^2 + 4\tau_{\theta\varphi}^2 = 4k^2 \qquad (k = \text{const}) \tag{2}$$

The plasticity condition [2] is valid for arbitrary plasticity states, since the third invariant of the stress deviator for the spherical state of deformation is zero.

Changing variables

$$\sigma_\theta = 2kp + k\cos 2\psi, \qquad \sigma_\varphi = 2kp - k\cos 2\psi, \qquad \tau_{\theta\varphi} = k\sin 2\psi \tag{3}$$

and substituting into the equation of equilibrium (2), we obtain

$$\begin{gathered}\frac{\partial p}{\partial \theta} - \sin 2\psi\frac{\partial \psi}{\partial \theta} + \frac{\cos 2\psi}{\sin\theta}\frac{\partial \psi}{\partial \varphi} + \cos 2\psi\cot\theta = 0\\ \frac{1}{\sin\theta}\frac{\partial p}{\partial \varphi} + \cos 2\psi\frac{\partial \psi}{\partial \theta} + \frac{\sin 2\psi}{\sin\theta}\frac{\partial \psi}{\partial \varphi} + \sin 2\psi\cot\theta = 0\end{gathered} \tag{4}$$

The system of equations (4) belongs to the hyperbolic type, and its characteristics are

$$\left(\frac{d\varphi}{d\theta}\right)_{1,2} = \frac{\tan(\psi \pm {}^1\!/_4\,\pi)}{\sin\theta} \tag{5}$$

Along the characteristics we have

$$dp \pm d\psi \pm \cos\theta\, d\varphi = 0 \tag{6}$$

The relations (5) and (6) can be represented in the form of finite differences and utilized for solving some concrete problems.

In the problem of the penetration of a rigid, plane wedge-like punch into a half-space (Fig. 1 represents the top view, BOB_1 corresponds to the contact surface of the punch, the areas BOC and B_1OC_1 are the plastic zones on the free surface of the half-space), we have to consider a rectangle

$$\frac{\pi}{2} \leqslant \theta \leqslant \pi \quad \left(0 \leqslant \theta \leqslant \frac{\pi}{2}\right), \qquad -\pi \leqslant \varphi \leqslant \pi$$

It is obvious from (5) and (6) that the change of the angle φ by a constant magnitude has no effect, it is only necessary that the length of the interval of φ was 2π. Because of symmetry it is sufficient to consider the interval $0 \leqslant \varphi \leqslant \pi$.

The solution of the problem was obtained on the computer Ural 1 VTs VGU. The results are shown in Fig. 2. In Fig. 2 the interval AC corresponds to the φ-segment equal to π. The segment BC corresponds to the half of the angle of the opening of the free surface of the half-space, AB corresponds to the half of the opening of the punch.

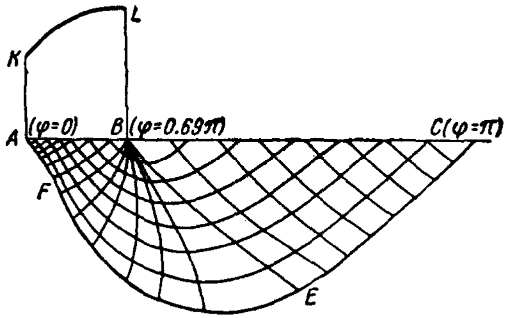

Fig. 2.

The solution in BCE can be obtained analytically [1]. In this case

$$p = -\ln\sin\theta - \frac{1}{2}$$

$$\varphi \pm \ln\tan\frac{\theta}{2} = \text{const} \qquad (7)$$

The integration proceeded along the characteristic BE determined from (7). In the region BEF the solution of a degenerate Goursat problem was constructed and in the region ABF the solution of a mixed problem.

The graphs KL in Fig. 2 represent the length of the limiting pressure on the contact surface for arbitrary radius ρ = const. traced from the apex of the wedge. The magnitude of the pressure is decreasing from the edge of the punch relative to the value determined from the Prandtl formula $q = -k \times (2 + \pi)$.

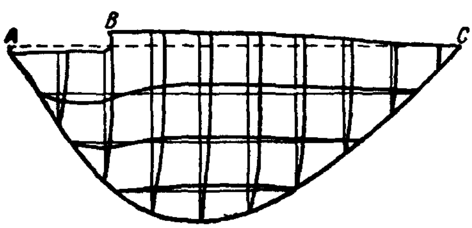

Fig. 3.

The maximum of the opening angle of the punch for which the given solution is applicable (angle BOB_1 in Fig. 1) is approximately equal to 1.4 or 80°. For large angles plastic zones of the material BOC and B_1OC_1 will overlap.

The values of the pressure q for various angles φ are given in the table below

φ =0	0.02	0.04	0.06	0.07	0.10	0.14	0.20	0.26	0.32	0.39	0.49	0.59	0.69
q =2,21	2.81	3.43	3.83	4.03	4.22	4.40	4.58	4.73	4.87	4.98	5.07	5.12	5.14

Clearly, the results given in the table above are applicable for small opening angles of the wedge as well.

The deformation velocities can be found from the relations along the characteristics [1]

$$\cos\theta\, d\varphi \left[v \tan\left(\psi \pm \frac{1}{4}\pi\right) - w\right] + dv + dw \tan\left(\psi \pm \frac{1}{4}\pi\right) = 0 \tag{8}$$

where v and w are deformation velocities along θ and φ, respectively.

The limiting conditions for the determination of the velocities is given by $v = 1$, *AB* in Fig. 2, the boundary *AFEC* divides the regions of plastic and rigid states of the material, and therefore the velocity normal to *AFEC* is zero. These conditions determine completely the construction of the velocity field in the region *AFECBA*.

The distortion of the coordinate net and the profile of the bulging surface *BC* during the penetrations of the punch are shown in Fig. 3.

If the opening angle BOB_1 of the wedge is larger than 80° (Fig. 4), then the solution may be obtained in the following way. It is only necessary to consider the rectangle $\pi/2 \leqslant \theta \leqslant \pi$ (or $0 \leqslant \theta \leqslant \pi/2$) and to take into account the symmetry of the solution, $0 \leqslant \varphi \leqslant 2\pi$.

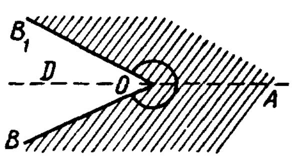

Fig. 4.

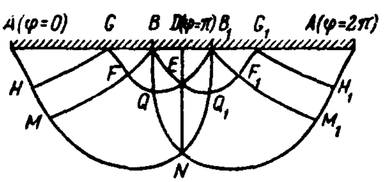

Fig. 5.

The halves *AB* and B_1A of the punch in the θ-plane are shown in Fig.5. Since the solution must be symmetrical with respect to the axis *DG*, we may limit our attention to the considerations of the region *AGBDENMHA*. Since along the line *DN* always $\tau_{0\varphi} = 0$, hence $\psi = 0$ (or $4 = \pi/2$) there, and consequently the unknown variables may be determined from (5).

The solution in the region *GBDEQFG* does not contain anything new which was not included in the solution of the previous problem.

In the region *QEN* a solution of the mixed problem, in the region *HFQNMH* a solution of the Goursat problem, and in the region *AGH* again of the mixed problem are sought.

The determination of the velocity fields are made for the following boundary conditions: $v = 1$ on *AB*, velocity normal to *AHMN* is zero; $w = 0$

on *DN*. In the region *ABMA* the mixed problem is solved, in *MBN* the Goursat problem and in *BQND* again the mixed problem is considered.

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